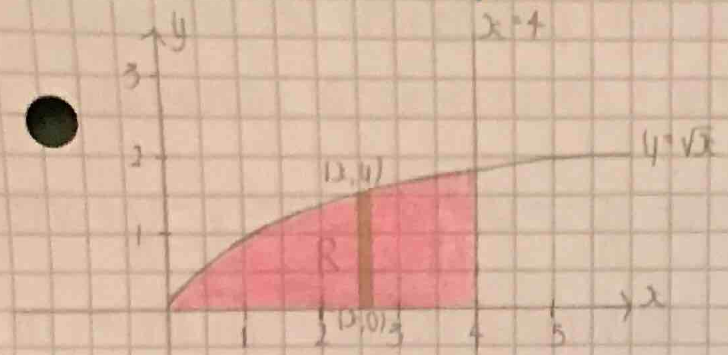
998: PART A

1)



a) Area R $= \int_0^4 \sqrt{x}\,dx =$

$= \left[\frac{2}{3}x^{3/2}\right]_0^4$

$= \frac{2}{3}(4^{3/2})$

$= \frac{2}{3}(8)$

$= \frac{16}{3} = 5.333$

b) $\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\left[\frac{2}{3}x^{3/2}\right]_0^h = \left[\frac{2}{3}x^{3/2}\right]_h^4$

$\frac{2}{3}h^{3/2} = \frac{2}{3}(4)^{3/2} - \frac{2}{3}h^{3/2}$

$\frac{4}{3}h^{3/2} = \frac{2}{3}(8)$

$\frac{4}{3}h^{3/2} = \frac{16}{3}$

$h^{3/2} = 4$

$h = 4^{2/3}$

$h = \sqrt[3]{16} = 2.520$ to 3.d.p.

c) $R(x) = y - 0 = y = \sqrt{x}$

volume $= \pi\int_0^4 (\sqrt{x})^2\,dx$

$= \pi\int_0^4 x\,dx$

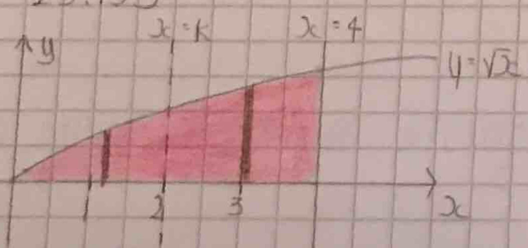
$= \pi\left[\frac{x^2}{2}\right]_0^4$

$= \pi\left(\frac{16}{2}\right)$

$= 8\pi$

$= 25.133$

d)



$\pi\int_0^k (\sqrt{x})^2\,dx = \pi\int_k^4 (\sqrt{x})^2\,dx$

$\pi\int_0^k x\,dx = \pi\int_k^4 x\,dx$

$\pi\left[\frac{x^2}{2}\right]_0^k = \pi\left[\frac{x^2}{2}\right]_k^4$

$\pi\left(\frac{k^2}{2}\right) = \pi\left(\frac{16}{2} - \frac{k^2}{2}\right)$

$\frac{\pi k^2}{2} = 8\pi - \frac{\pi k^2}{2}$

$\pi k^2 = 8\pi$

$k^2 = 8$

$k = \sqrt{8} = 2.828$

2) $f(x) = 2xe^{2x}$

a) $\lim_{x\to-\infty} f(x) = \frac{2(\infty)}{e^{-\infty}} = 0$

$\lim_{\lim\to\infty} f(x) = 2(\infty)e^{\infty} = \infty$ or doesn't exist

b) $f'(x) = 2e^{2x} + 4xe^{2x}$

$2e^{2x} + 4xe^{2x} = 0$

$2e^{2x}(1 + 2x) = 0$

$2e^{2x} \neq 0 \qquad 1 + 2x = 0$

$2x = -1$

$x = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)e^{2(-1/2)}$

$= -e^{-1}$

$= -\frac{1}{e} = -0.368$

$f'(x)$: $-$ | $+$ at -0.5

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$-\frac{1}{e}$ is an absolute minimum value because:

c $f(x)$ changes sign from $-$ve to $+$ve at $x = -0.5$

$f'(x) < 0$ for $x < -0.5$

$f'(x) > 0$ for $x > -0.5$

and $x = -0.5$ is only critical number

c) Range $= -\frac{1}{e} \leq x < \infty$

d) $y = bxe^{bx}$

$y' = be^{bx} + b^2xe^{bx}$

$be^{bx} + b^2xe^{bx} = 0$

$be^{bx}(1 + bx) = 0$

$be^{bx} \neq 0, \quad 1 + bx = 0$

$bx = -1$

$x = -\frac{1}{b}$

$y = b\left(-\frac{1}{b}\right)e^{b(-1/b)}$ $\therefore$ y has an absolute minimum value of $-\frac{1}{e}$ for all nonzero b

$= -e^{-1} = -\frac{1}{e}$

a) Acceleration is positive on (0, 35) and (45, 50) because velocity is increasing on [0, 35] and [45, 50]

b) Average acceleration $= \frac{V(50) - V(0)}{50 - 0} = \frac{72 - 0}{50} = \frac{72}{50}$ ft/sec^2

c) $\frac{V(45) - V(35)}{45 - 35} = \frac{60 - 81}{10} = \frac{-21}{10}$ ft/sec^2

d) $\int_0^{50} V(t)\,dt = \frac{50-0}{5}\left[V(5) + V(15) + V(25) + V(35) + V(45)\right]$

$= 10(12 + 30 + 70 + 81 + 60)$

$= 2530$ feet

The integral is the total distance traveled in feet over the time 0 to 50 seconds

18: PART B

4) $f(1)=4,\ f'(x)=\dfrac{3x^2+1}{2y}$

a) $f'(1)=\dfrac{3(1)+1}{2(4)}=\dfrac{4}{8}=\dfrac{1}{2}$

b) Equation of tangent:

$y-4=\dfrac{1}{2}(x-1)$

$f(1.2)-4=\dfrac{1}{2}(1.2-1)$

$f(1.2)=\dfrac{0.2}{2}+4$

$f(1.2)=0.1+4=4.1$

c) $\dfrac{dy}{dx}=\dfrac{3x^2+1}{2y},\ f(1)=4$

$\int 2y\,dy=(3x^2+1)dx$

$y^2=x^3+x+c$

$4^2=1+1+c$

$c=14$

$y^2=x^3+x+14$

$y=\sqrt{x^3+x+14}$

$f(x)=\sqrt{x^3+x+14}$

d) $f(1.2)=\sqrt{1.2^3+1.2+14}=4.114$

5) $F(t)=80-10\cos\left(\dfrac{\pi t}{12}\right),\ 0\le t\le 24$

b) Average Temp $=\dfrac{1}{14-6}\int_6^{14}F(t)\,dt$

$=\dfrac{1}{8}\int_6^{14}\left(80-10\cos\left(\dfrac{\pi t}{12}\right)\right)dt$

$=87.162$

$=87°F$

c) $80-10\cos\left(\dfrac{\pi t}{12}\right)\ge 78$

$0\le t\le 24$

$0\le \dfrac{\pi t}{12}\le 2\pi$

$80-10\cos\left(\dfrac{\pi t}{12}\right)-78\ge 0$

$2-10\cos\left(\dfrac{\pi t}{12}\right)\ge 0$

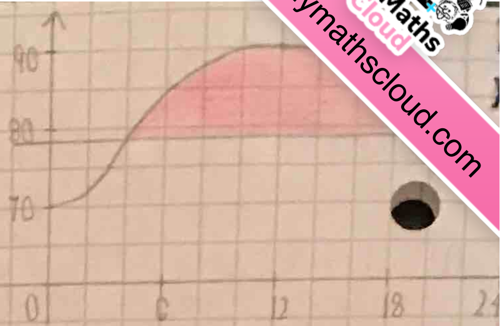
$\cos\left(\dfrac{\pi t}{12}\right)=\dfrac{1}{5}$

$\dfrac{\pi t}{12}=1.36944,\ 4.91374\ \Rightarrow t=5.231,\ 18.770$

$5.231\le t\le 18.770$

d) Total cost $= 0.05\int_{15.231}^{18.770}\left(80 - 10\cos\left(\frac{\pi t}{12}\right) - 78\right)dt$

$= 0.05(101.92741) = 5.096 \approx \$5.10$



5) $2y^3 + 6x^2y - 12x^2 + 6y = 1$

a) $6y^2\frac{dy}{dx} + 12xy + 6x^2\frac{dy}{dx} - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$6\frac{dy}{dx}(y^2 + x^2 + 1) = 24x - 12xy$

$\frac{dy}{dx} = \frac{4x - 2xy}{y^2 + x^2 + 1}$

b) $\frac{dy}{dx} = 0$

$\frac{4x - 2xy}{y^2 + x^2 + 1} = 0$

$4x - 2xy = 0$

$2x(2 - y) = 0$

$x = 0,\ y = 2$

when $x = 0$: $2y^3 + 6y = 1$

$y = 0.165$

when $y = 2$: $16 + 12x^2 - 12x^2 + 12 = 1$

$28 \neq 1$ $\therefore$ there is no point on the curve with y coordinate of 2

Equ. of Tangent:

$y - 0.165 = 0(x - 0)$

$y = 0.165$

$y = 0.165$ is the equation of the only horizontal tangent line

c) $y - 0 = -1(x - 0)$

$y = -x$

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-2x^3 - 6x^3 - 12x^2 - 6x = 1$

$-8x^3 - 12x^2 - 6x = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -\frac{1}{2},\ y = \frac{1}{2}$

or:

$\frac{dy}{dx} = -1$

$\frac{4x - 2xy}{x^2 + y^2 + 1} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x - 2x(-x) = -x^2 - (-x)^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0 \Rightarrow x = -1/2,\ y = 1/2$